



Blockmodels and Allegiance

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Agenda

- Blockmodel
- Necessity
- Allegiance
- Toy Example
- Real Example
- Conclusions

Acknowledgments

- This work is supported by the In-house Laboratory Independent Research (ILIR) program.

Blockmodel

- D - directed graph
- E - edges(ties)
- V - vertexes(actors)
- $P = \{P_1, \dots, P_k\}$ - Partitions
- $B = \{B_{1,1}, B_{1,2}, \dots, B_{n,n-1}, B_{n,n}\}$ - Blockmodel
 - $B_{i,j}$, when $i \neq j$ - ties from actors in partition i to partition j
 - $B_{i,j}$, when $i = j$ - internal ties between actors in partition i

Necessity

- Measure blockmodel strength change
- Estimate the true number of partitions
- Turn the time separated graphs into a single graph

Allegiance

- is the measure of how much an actor is helping his block
- Help:
 - Having internal edges to other block members
 - $H_{int}(actor) = \sum_{i=N_{P_i}} E(actor)$
 - Not having external edges to non-blockmembers
 - $H_{ext}(actor) = N - N_{P_i} - \sum_{i \neq N_{P_i}} E(actor)$

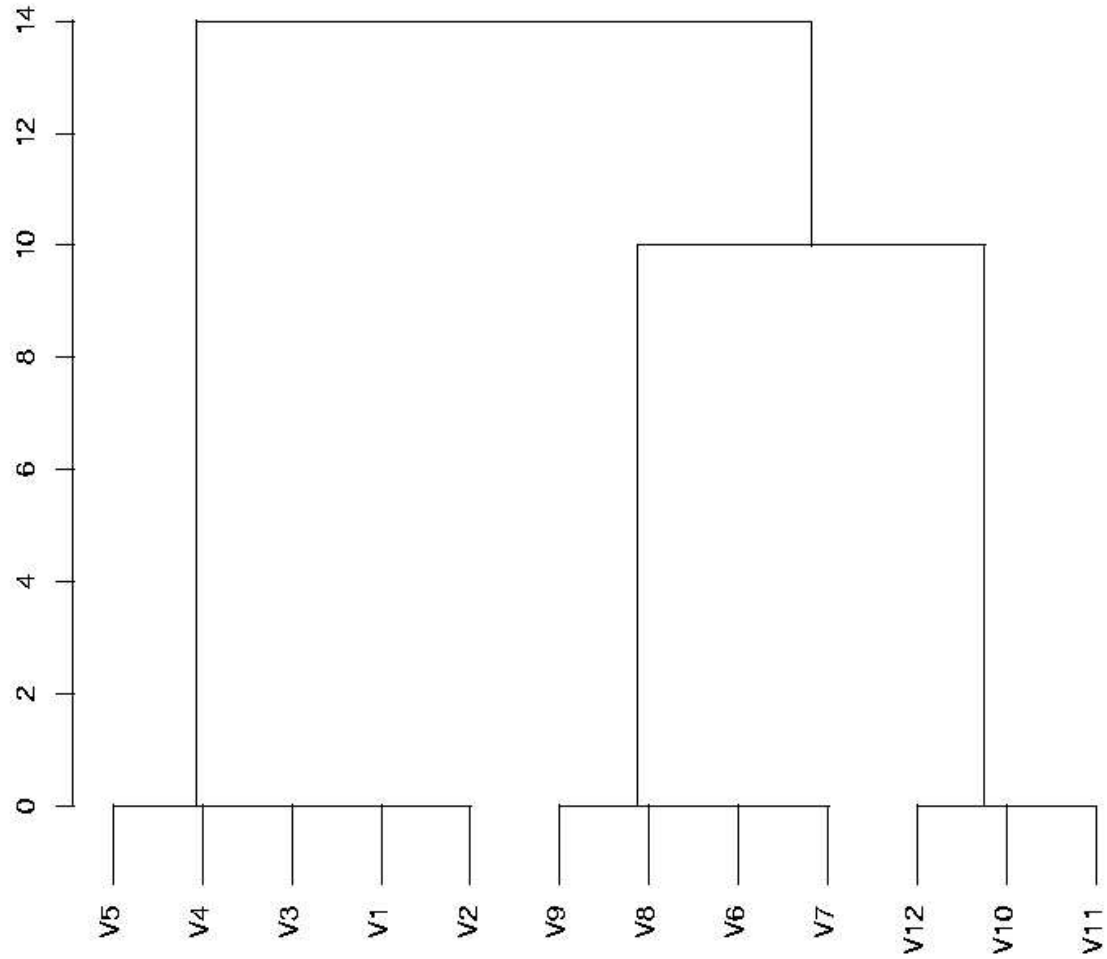
Allegiance Equation

- $A(i) = \alpha[H_{int}(i)] + (1 - \alpha)[H_{ext}(i)]$
- $\alpha \in [0, 1]$ - weight function

Toy Example

	V5	V4	V3	V1	V2	V9	V8	V6	V7	V12	V10	V11
V5	█	█	█	█	█							
V4	█	█	█	█	█							
V3	█	█	█	█	█							
V1	█	█	█	█	█							
V2	█	█	█	█	█							
V9						█	█	█	█			
V8						█	█	█	█			
V6						█	█	█	█			
V7						█	█	█	█			
V12										█	█	█
V10										█	█	█
V11										█	█	█

Equivalence Cluster



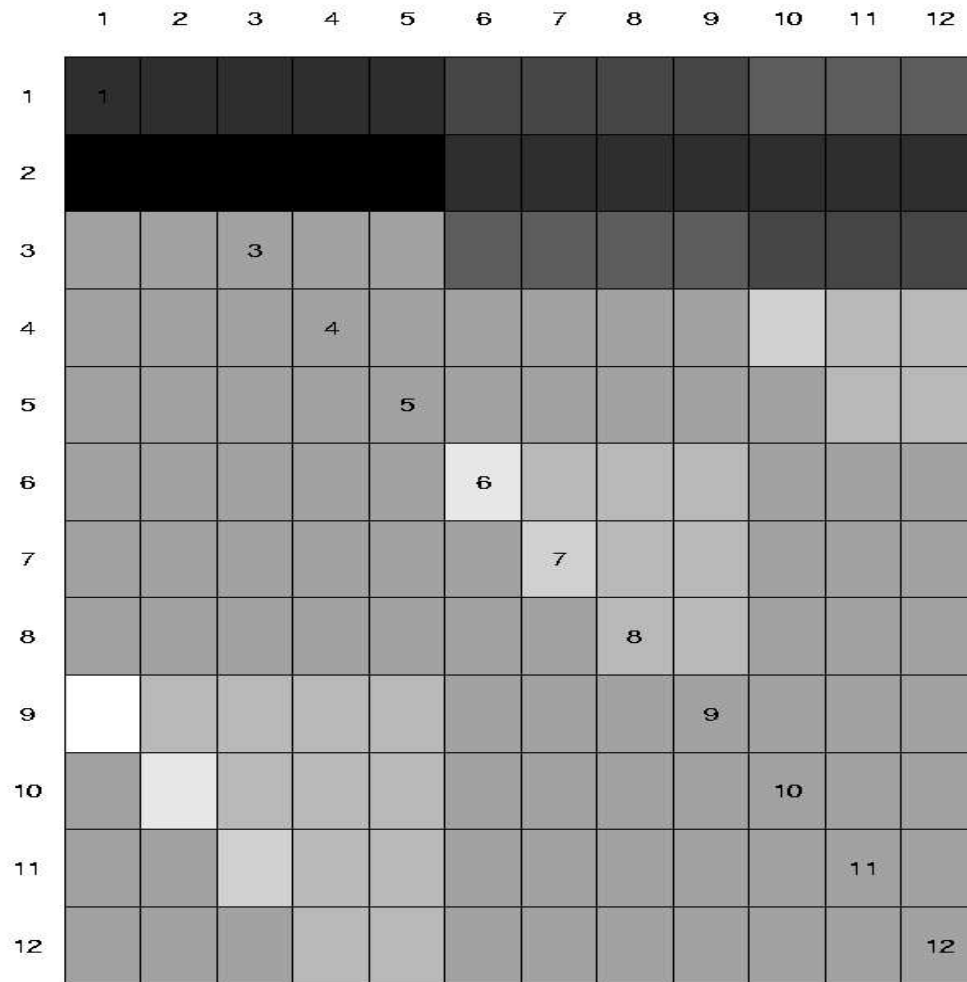
k = 4

	V5	V4	V3	V1	V2	V9	V8	V6	V7	V12	V10	V11
V5	█											
V4	█											
V3	█											
V1	█											
V2	█											
V9						█						
V8						█						
V6						█						
V7						█						
V12										█		
V10										█		
V11										█		

Measure Change in Allegiance

- $DA_{(i,k)} = A_{i,k} - A_{i,k-1}$ for $k > 1$
- At $k = 1$
 - $DA_{(i,1)} = A_{i,1}$
 - Out degree of actor i

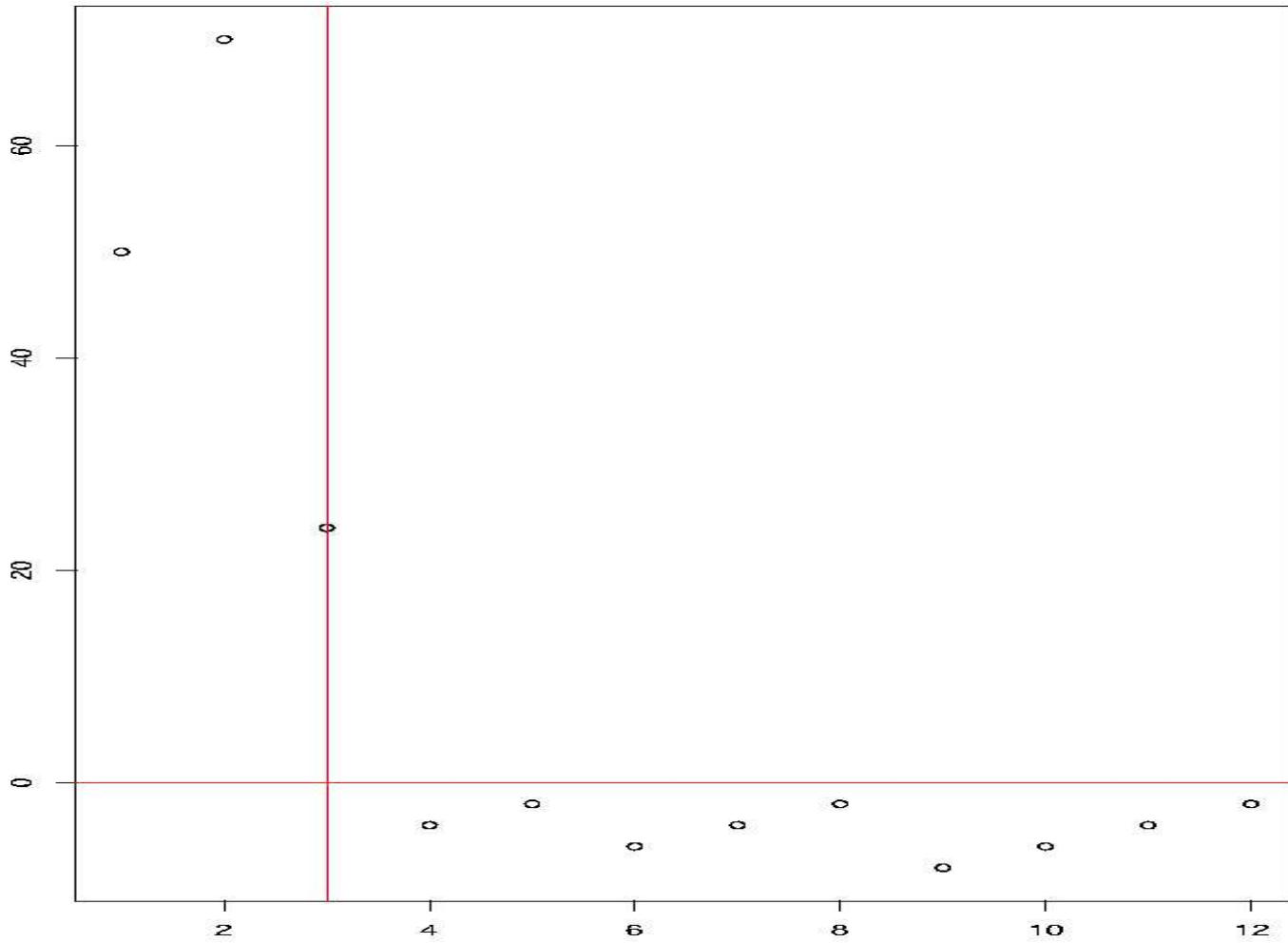
Plot of Change in Allegiance



Sum of Allegiance Change

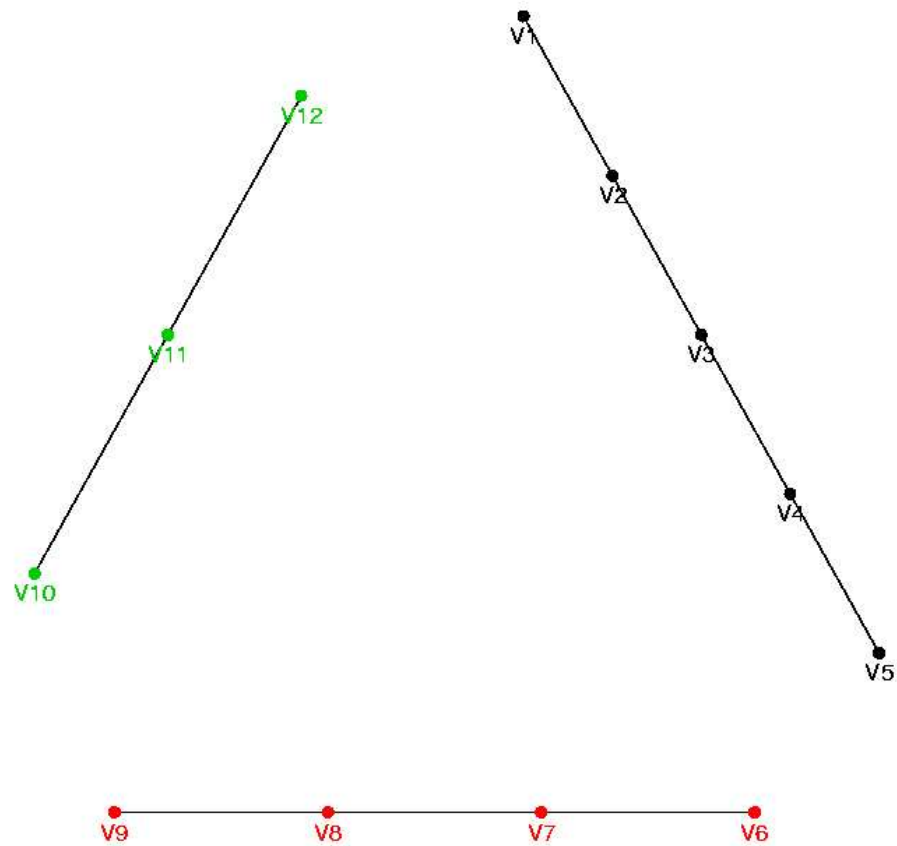
- $SDA_k = \sum_{i=1}^n DA_{(i,k)}$
- Used to estimate k : the number of partitions

Plot of SDA_k



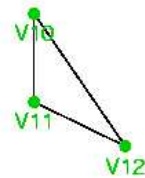
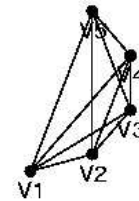
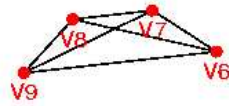
$k = 3$ mpoly graph

Toy Problem K=3



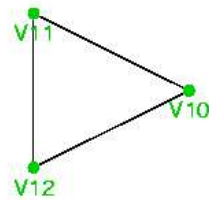
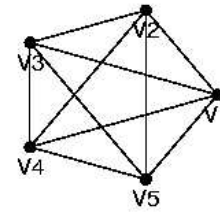
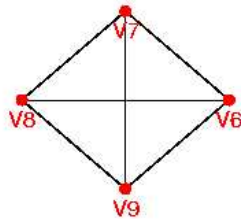
$k = 3$ mflower graph

Toy Problem K=3



$k = 3$ pinwheel graph

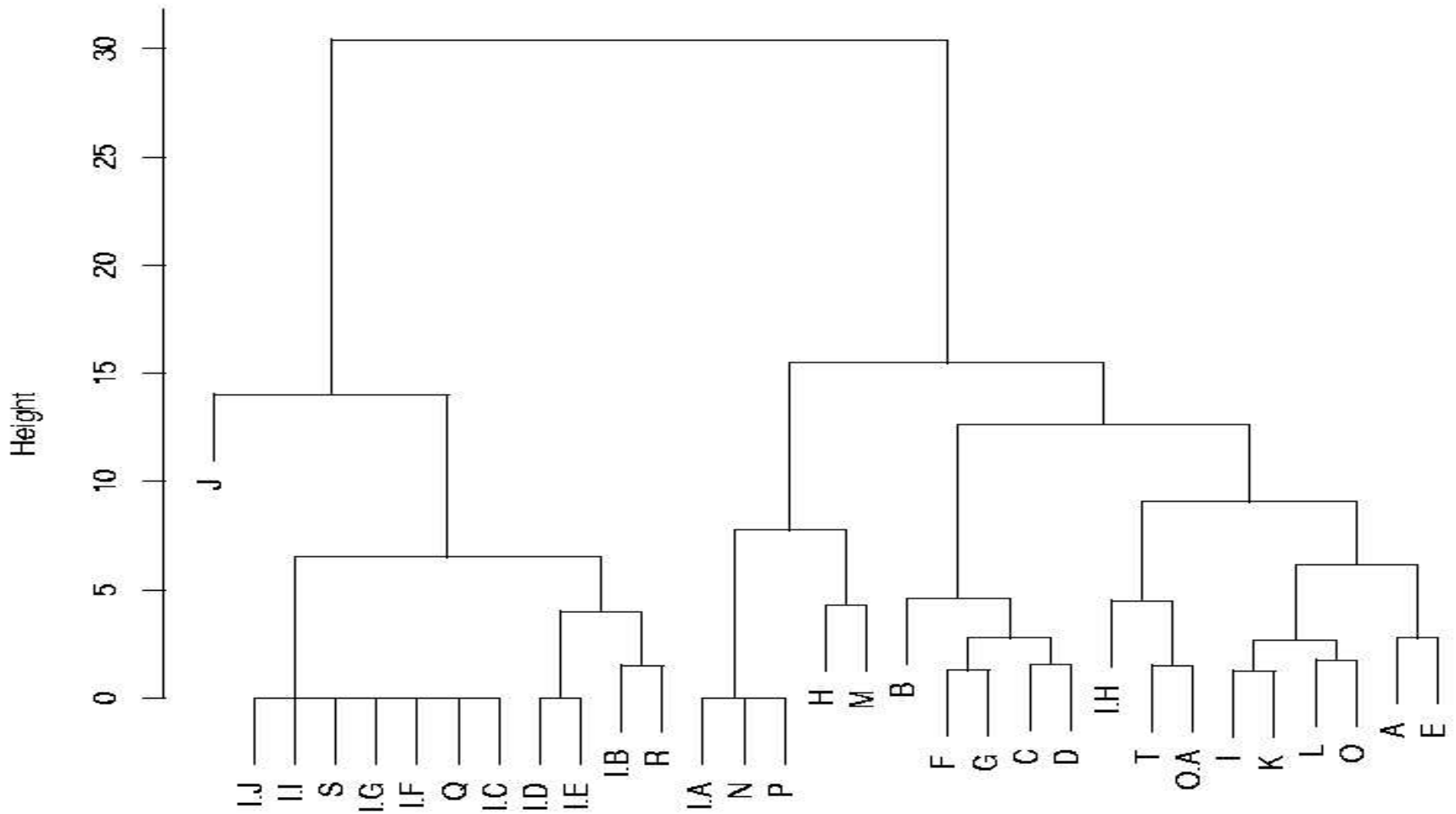
Toy Problem K=3



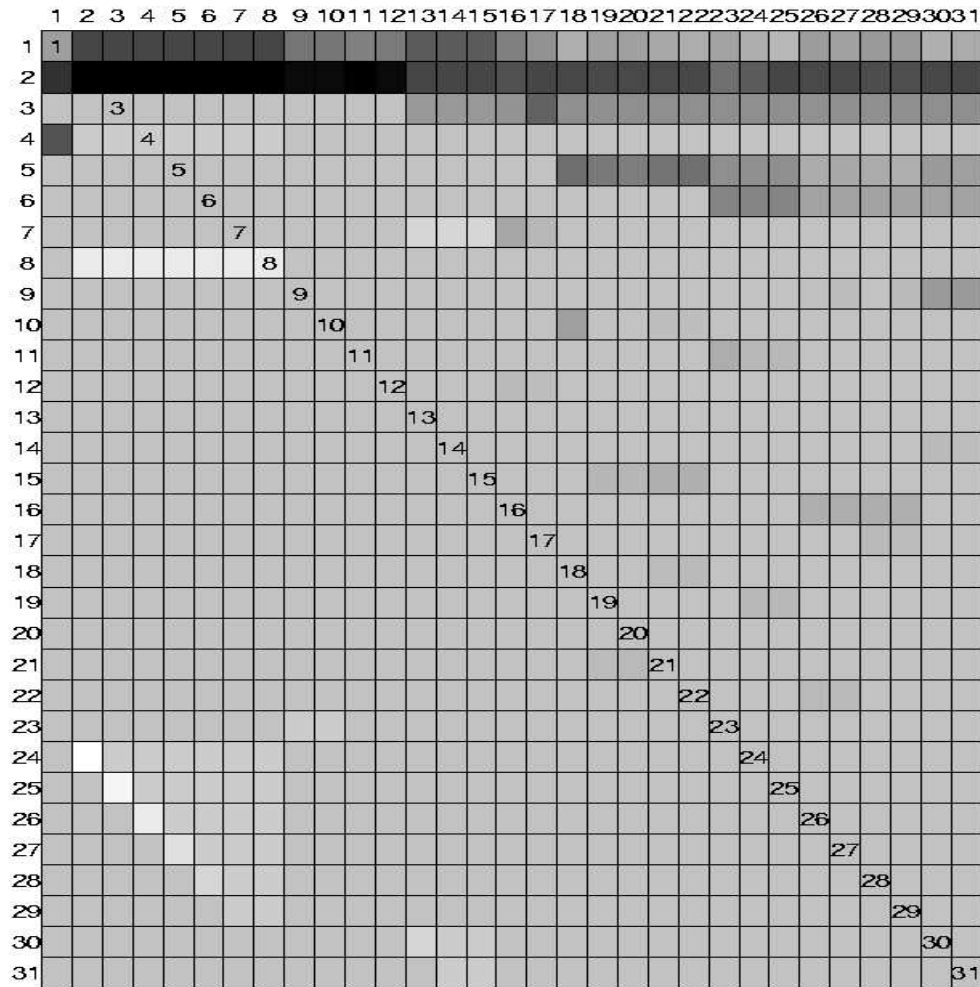
Real Example

- 5 users
- Oct 2002 - Feb 2004
- 3000 total servers accessed over 14 months
- 31 total clients
- 19 LAN computers

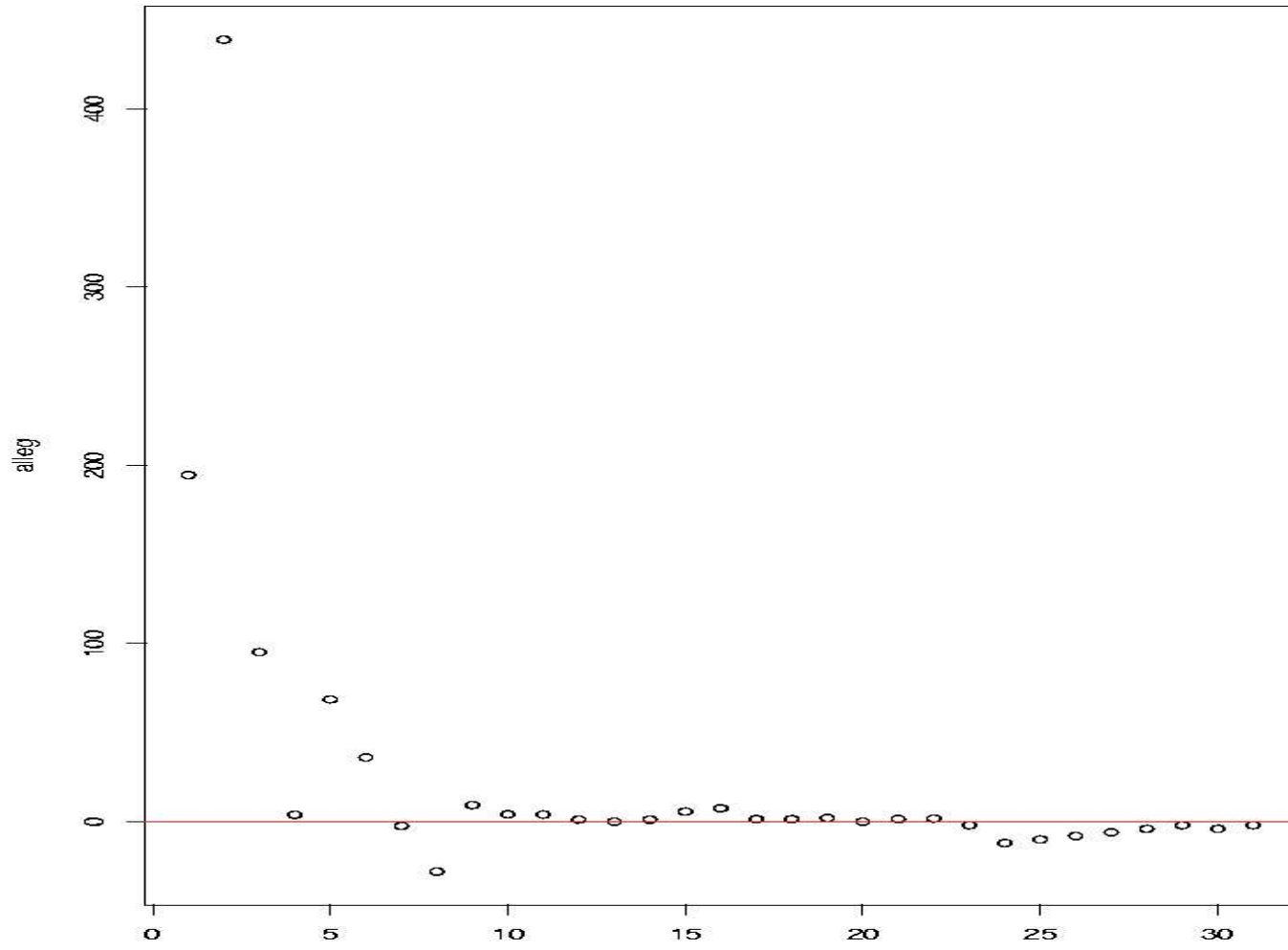
Equivalence Cluster



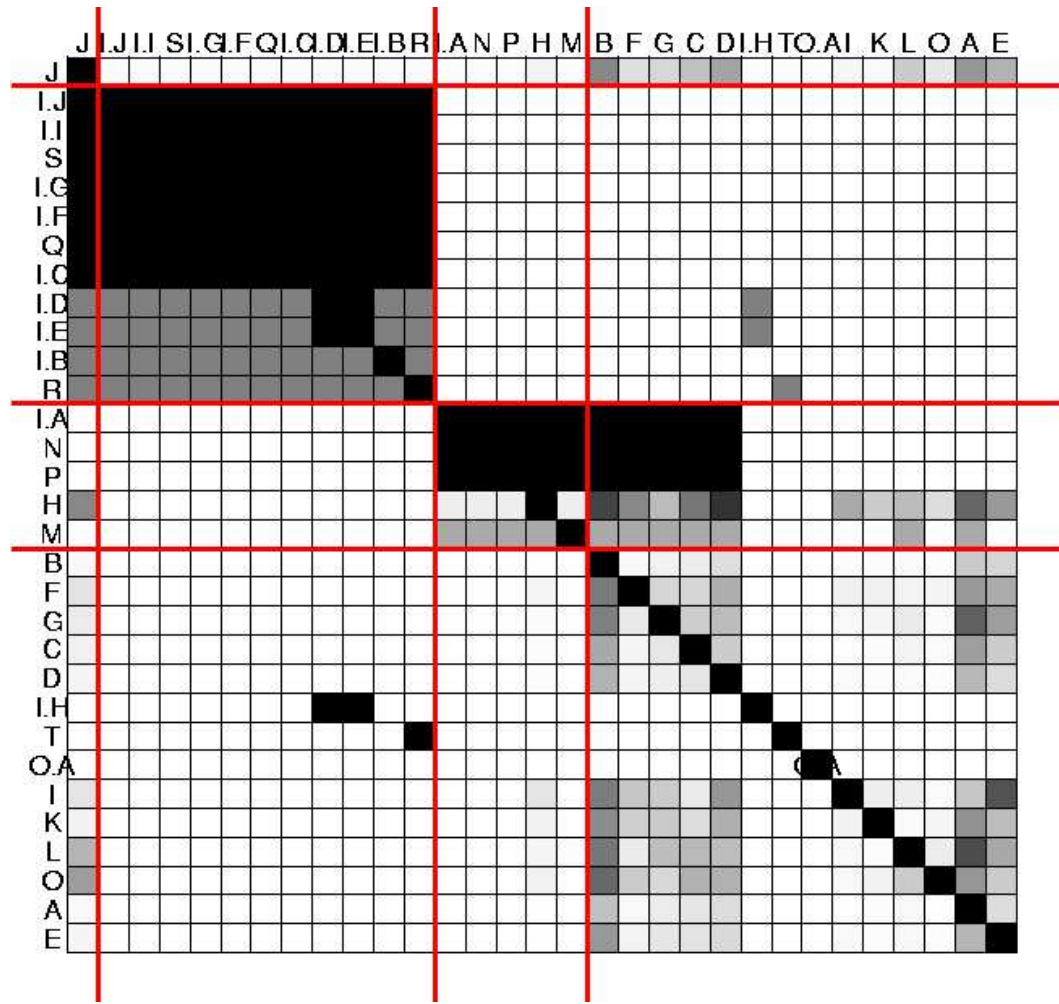
Individual allegiance Change



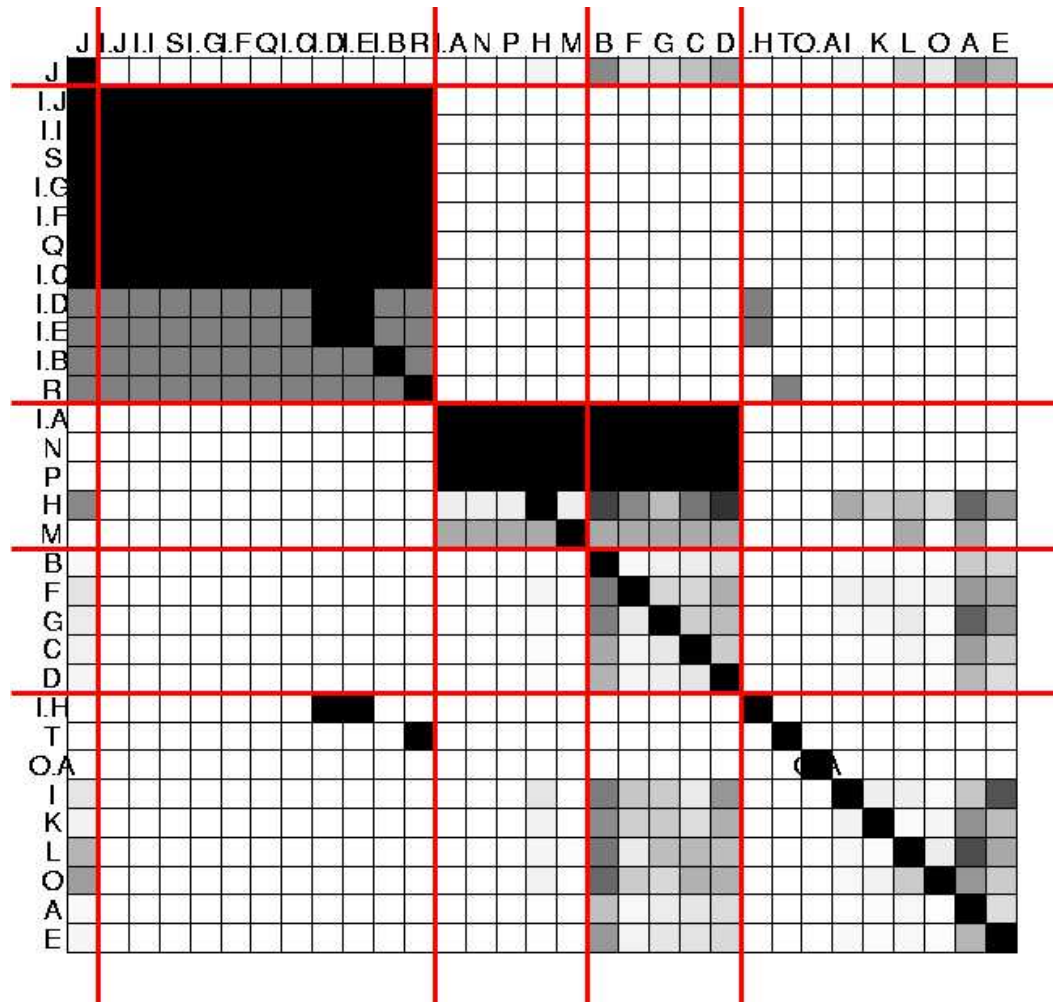
Total Allegiance Change



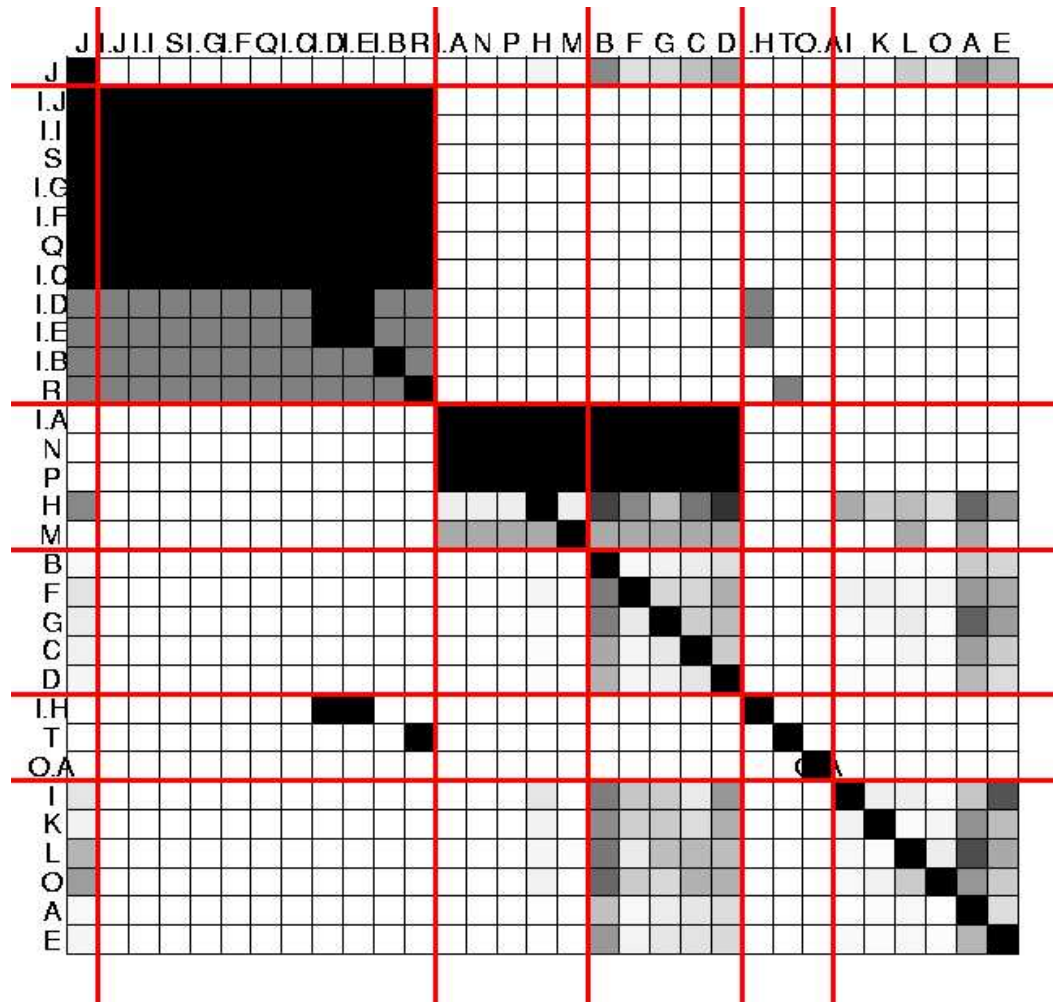
k = 4



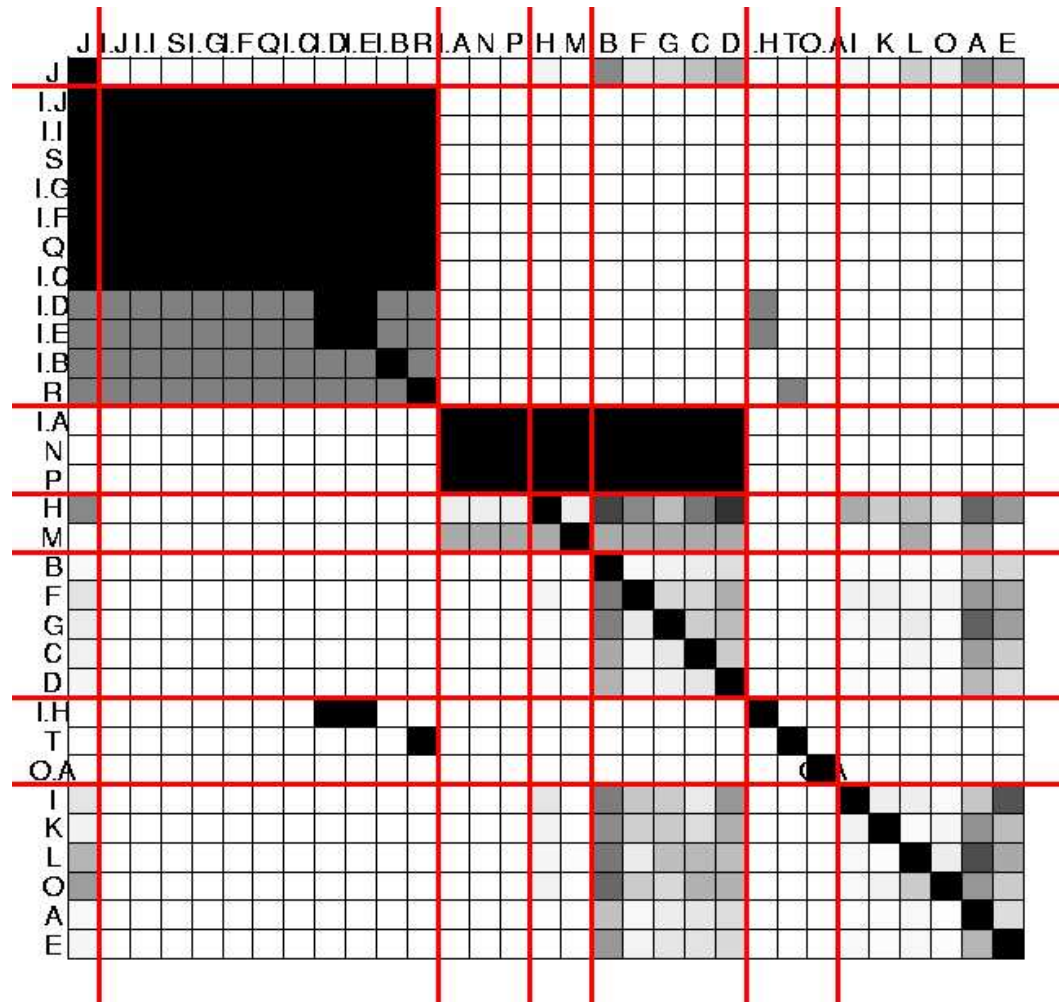
$k = 5$



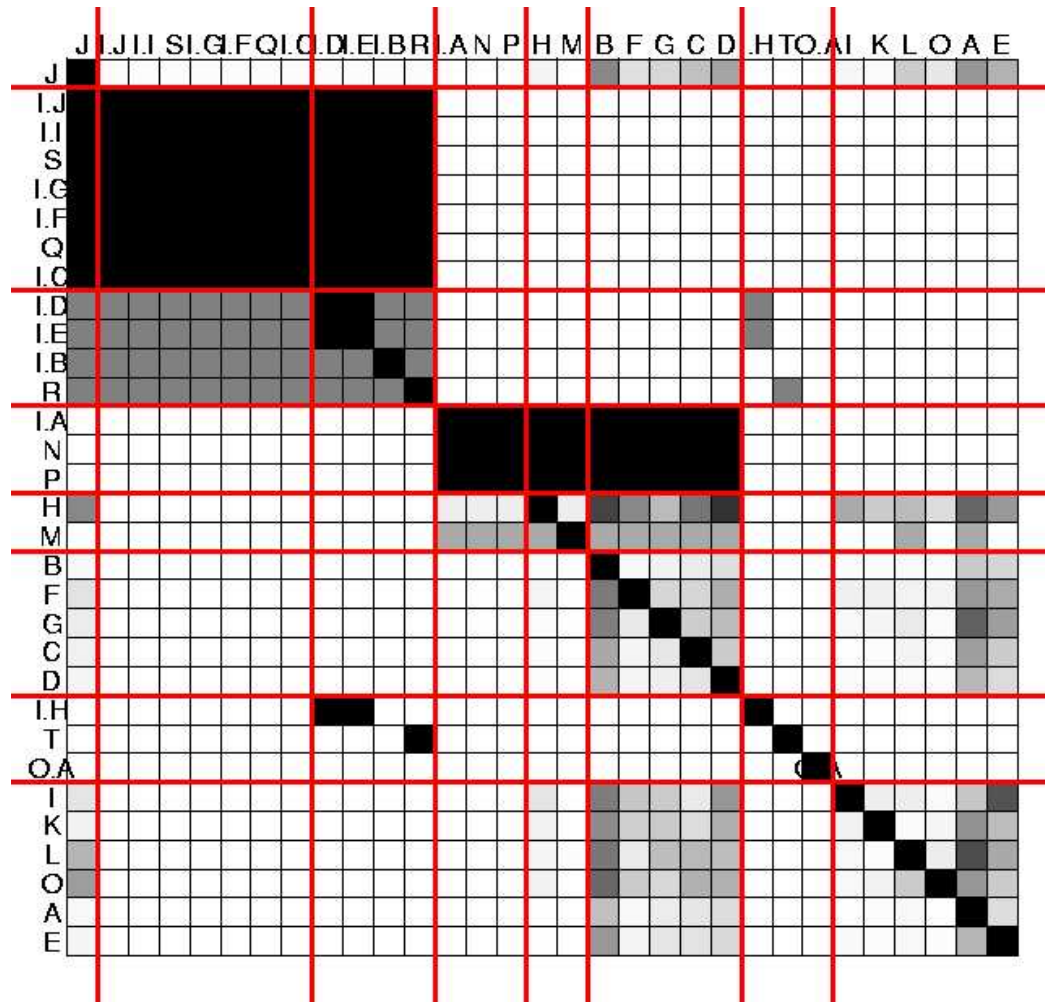
$k = 6$



$k = 7$

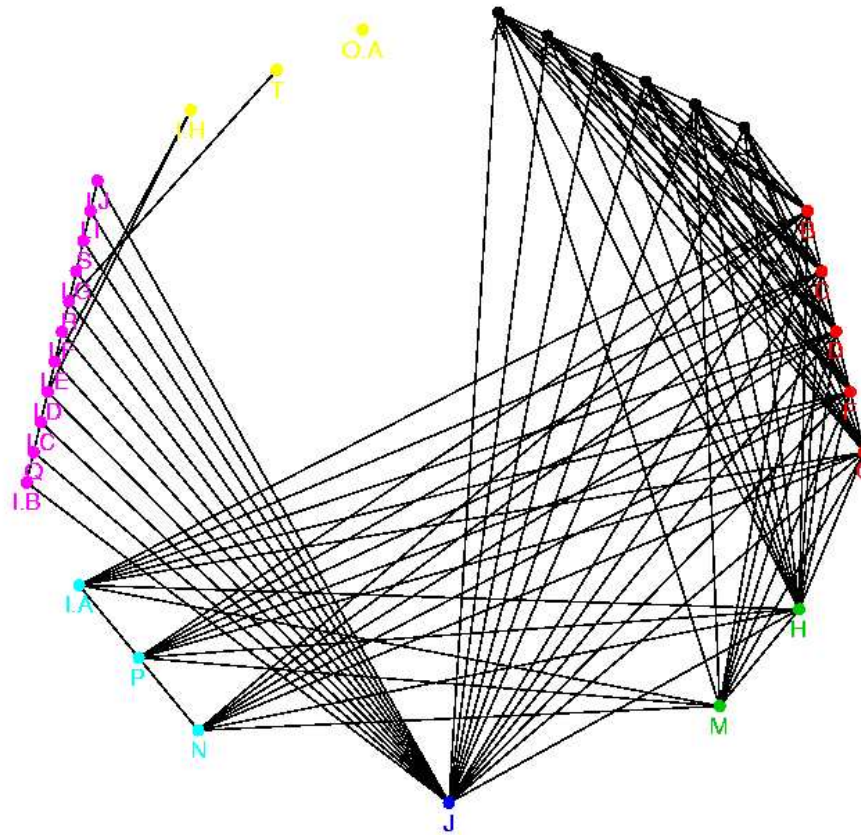


k = 8



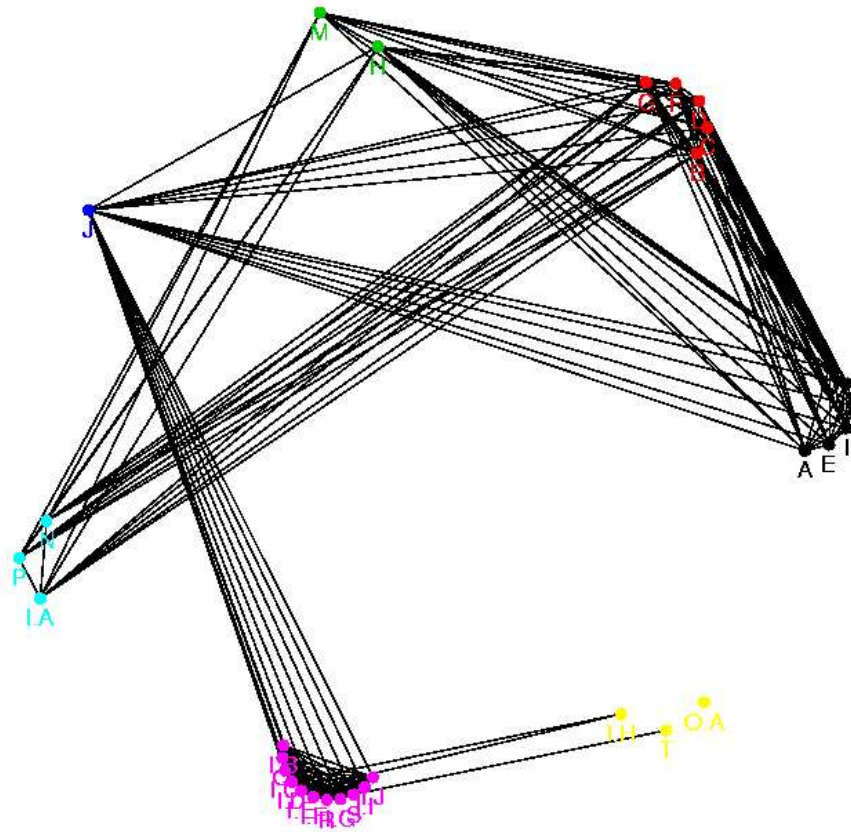
$k = 7$ mpoly graph

Real Network Problem K=7



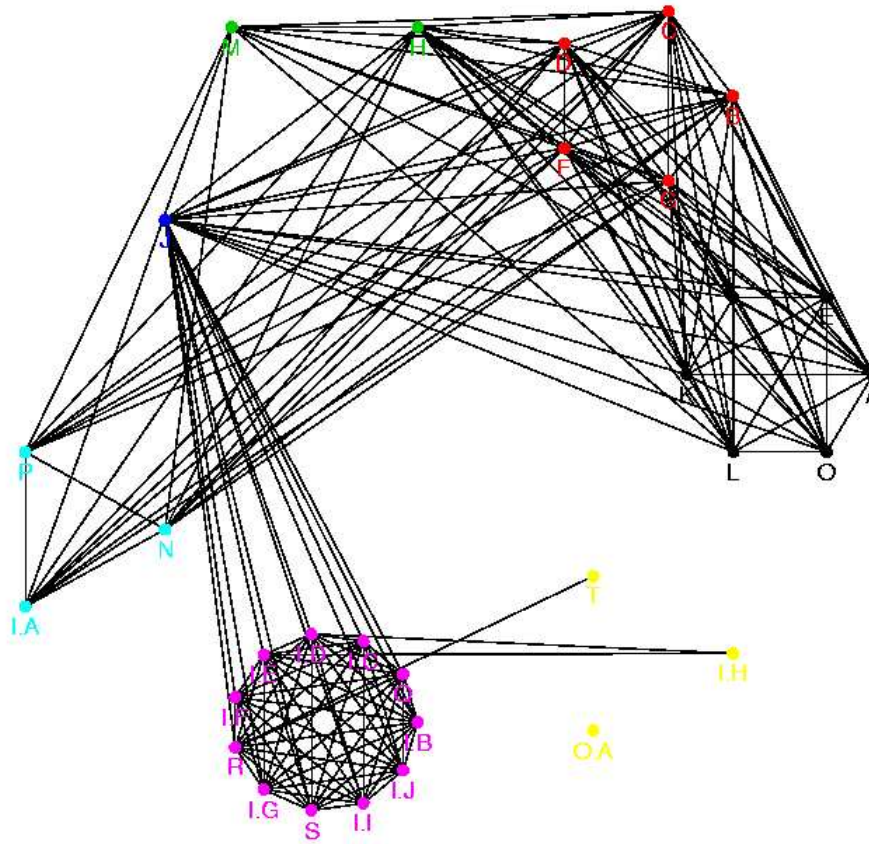
$k = 7$ mflower graph

Real Network Problem K=7



$k = 7$ pinwheel graph

Real Network Problem K=7



Conclusions

- Allegiance
 - Can be used to estimate number of partitions
 - Used to estimate maximum number of partitions at a minimum
 - Gives a measure for block strength change as partition size changes